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LARGE ANGLE METHOD FOR SPACE VEHICLE ANGULAR MOMENTUM DESATURATION USING GRAVITY GRADIENT TORQUES

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16. ABSTRACT
The angular momentum desaturation method is based on the use of the gravity gradient torques. Large angular maneuvers maximize the effectiveness of the desaturation method for the case where the mission requires the minimum moment-of-inertia axis to point toward some celestial object for the nonocculted part of the orbit. The desaturation method is developed under the assumption that the difference between two of the principal moments of inertia is negligible. A constant attitude with respect to the gravity gradient direction is held during the desaturation period. This allows maximization of the available gravity gradient desaturation torque which is necessary since the assumed moment-of-inertia distribution (in relation to the mission requirements) is a very poor candidate for angular momentum desaturation using the gravity gradient, especially when relatively large aerodynamic disturbance torques are present.

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DEFINITION OF SYMBOLS

A_i	$= 2/(3\Omega\ \Delta I_x)$
c	cosine
$\underline{H}$	angular momentum (Nms)
$H_{x,y,z}$	T-components of the desaturation momentum (Nms)
$\underline{H}_A$	average day momentum (Nms)
$\underline{H}_B$	bias momentum (Nms)
$\underline{H}_D$	momentum to be desaturated (Nms)
$\Delta\underline{H}_D$	change in $\underline{H}_D$ (Nms)
$\underline{H}_{max}$	maximum day momenta (Nms)
$\underline{H}_{min}$	minimum day momenta (Nms)
$\underline{H}_o$	initial angular momentum (Nms)
I	moment-of-inertia matrix (kg m^2)
$I_{x,y,z}$	principal moments of inertia (kg m^2)
$\Delta I_{x,y,z}$	principal moment of inertia differences (kg m^2)
k	small positive quantity
K_{ij}	gains
K_n	gain
$K_{(n-1)}$	gain
K_ϵ	$\Omega\ \dot{\phi}_L$
n, (n-1)	subscript referring to present (n) or past orbit (n-1)
$\underline{r}$	unit vector parallel to the gravity gradient vector

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$\underline{r}$	unit vector parallel to the gravity gradient vector

DEFINITION OF SYMBOLS (Continued)

$r_{x,y,z}$	components of $\underline{r}$ in the P-system
s	sine
$\underline{T}_G$	gravity gradient torque (Nm)
$\underline{T}_{GP}$	gravity gradient torque (components in the P-system) (Nm)
$\underline{T}_{GI}$	gravity gradient torque (components in the I-system) (Nm)
$\dot{\epsilon}_{ij}$	maneuver rates (about vehicle axes) (rad/s)
η_x, η_t	see Appendix A for definition (rad)
$\Delta\eta_t$, $\Delta\eta_{tL}$, μ_x, μ_{xL}, μ_y, μ_{yL}, $\Delta\mu_y$, μ_z, μ_{zL}	maneuver angles and their respective limits (rad)
ρ	orbital half angle used for desaturation (rad)
$\dot{\sigma}_L$	vehicle angular rate limit (rad/s)
Ω	orbital angular velocity (rad/s)

DEFINITION OF SYMBOLS (Continued)

$$[\eta_x] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\eta_x & s\eta_x \\ 0 & -s\eta_x & c\eta_x \end{bmatrix}$$

$$[\eta_y] = \begin{bmatrix} c\eta_y & 0 & -s\eta_y \\ 0 & 1 & 0 \\ s\eta_y & 0 & c\eta_y \end{bmatrix}$$

$$[\eta_z] = \begin{bmatrix} c\eta_z & s\eta_z & 0 \\ -s\eta_z & c\eta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

In general, angular quantities in brackets with a subscript indicate a rotation matrix about the axis of the subscript. Other examples are $[\lambda_y]$, $[\lambda_z]$, $[\Phi_z]$, $[\mu_x]$, $[\mu_y]$, $[\mu_z]$

$$[\eta_t] = \begin{bmatrix} c\eta_t & 0 & -s\eta_t \\ 0 & 1 & 0 \\ s\eta_t & 0 & c\eta_t \end{bmatrix}$$

$$[\theta] = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}$$

DEFINITION OF SYMBOLS (Concluded)

$$[\kappa] = \begin{bmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{bmatrix}$$

TECHNICAL MEMORANDUM X-53958

LARGE ANGLE METHOD FOR SPACE VEHICLE ANGULAR MOMENTUM DESATURATION USING GRAVITY GRADIENT TORQUES

SUMMARY

An angular momentum desaturation method is presented for space vehicles in orbit. This method utilizes the gravity gradient torques and therefore avoids the necessity for mass expulsion by reaction jet attitude control systems. The desaturation method is based on the assumption that the minimum moment-of-inertia axis has to be aligned with the sun vector during orbital daylight for solar experiments. It is further assumed that the difference between the two large moments of inertia is negligible. Large angular maneuvers of the minimum moment-of-inertia axis during the orbital night maximize the gravity gradient torques that are then utilized for desaturation of the angular momentum stored by a momentum storage device such as control moment gyros.

The basic idea of the desaturation method is that, for as long as possible, the vehicle minimum moment-of-inertia axis keeps a constant attitude with respect to a rotating coordinate system in which the local vertical is along one of the axes and the orbital angular velocity is along another. The constant attitude, with respect to the rotating coordinate system, is calculated for proper momentum desaturation.

To keep a constant attitude in a system rotating with constant angular velocity requires, in general, inertial angular rates about all body axes simultaneously. These rates are constant and present no problem for the implementation of the desaturation method.

The gravity gradient torque is perpendicular to both the local vertical and the minimum moment-of-inertia axis for the assumed vehicle configuration.

INTRODUCTION

Some missions for space vehicles in orbit require an investigation of a celestial object which in turn requires an inertially fixed attitude during the time the celestial object is not occulted by the earth; e.g., the Apollo Telescope Mount (ATM) and its solar experiments [1,2]. In the following, ATM nomenclature and definitions will be used; e.g., sun instead of celestial object or night instead of occultation time, etc. The once-considered ATM/LM/CSM configuration will also be used, since it had the moment-of-inertia distribution typical for the angular momentum desaturation method under discussion.

Gravity gradient, aerodynamic, and other external torques acting on the vehicle during the time the vehicle is inertially fixed must be absorbed by an angular momentum storage device, which always has limited capacity. A method for momentum desaturation which does not require mass expulsion is desired. The fact that an inertially fixed attitude is not required during night allows the utilization of the gravity gradient in conjunction with vehicle attitude maneuvers to desaturate the momentum storage device.

Two typically different cases can be identified for the problem of momentum desaturation if it is assumed that two of the principal moments of inertia of the vehicle are about the same and the third is smaller. In the first case, the minimum moment-of-inertia axis can be put into the orbital plane and kept perpendicular to the sun vector. This case is treated in Reference 3 and does not present a problem. In the second case, the minimum moment-of-inertia axis is parallel to the sun vector, and a desaturation method for this case is treated in Reference 4. However, simulation showed that this method (which worked satisfactorily for the gravity gradient torques alone) was not effective when significant aerodynamic torques were encountered. The method described in the following sections increases the effectiveness by maximizing the gravity gradient torques and the time these torques are applied. This requires the minimum moment-of-inertia axis to have a constant attitude with respect to the gravity gradient direction, and this attitude must be maintained as long as possible.

GRAVITY GRADIENT TORQUE AND MOMENTUM ACCUMULATION

The gravity gradient torque acting on the vehicle can be expressed as

$$\underline{T}_G = 3\Omega^2 \tilde{r}\, I\, \underline{r} \tag{1}$$

for a vehicle in circular orbit (which is assumed for further development), where I is the vehicle moment-of-inertia matrix, Ω is the orbital rate, $\underline{r}$ is a unit vector parallel to the radius vector from the earth center to the vehicle center of mass, and $\tilde{r}$ is defined as

$$\tilde{r} = \begin{bmatrix} 0 & -r_z & +r_y \\ +r_z & 0 & -r_x \\ -r_y & +r_x & 0 \end{bmatrix} . \tag{2}$$

When equation (1) is developed for the principal moment-of-inertia system, P, we have

$$\underline{T}_{GP} = 3\Omega^2 \begin{bmatrix} 0 & -r_z & +r_y \\ +r_z & 0 & -r_x \\ -r_y & -r_x & 0 \end{bmatrix} \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{3}$$

or

$$\underline{T}_{GP} = 3\Omega^2 \begin{bmatrix} \Delta I_x r_y r_z \\ \Delta I_y r_z r_x \\ \Delta I_z r_x r_y \end{bmatrix} \tag{4}$$

with

$$
\begin{aligned}
\Delta I_x &= I_z - I_y \\
\Delta I_y &= I_x - I_z \\
\Delta I_z &= I_y - I_x
\end{aligned}
\tag{5}
$$

where I_x, I_y, I_z are principal moments of inertia, and the components of $\underline{r}$ are in the P-system. For convenience and comparison, the definitions for ATM coordinate systems are used which are given in Appendix A. Equation (4) can be further simplified if the assumption is made that I_x and I_y are identical:

$$
\underline{T}_{GP} = 3\,\Omega^2\,\Delta I_x\, r_z \begin{bmatrix} r_y \\ -r_x \\ 0 \end{bmatrix} . \tag{6}
$$

Throughout this report, the assumption is made that there is no misalignment between the vehicle principal moment-of-inertia axis and the geometric axes. Under this condition the components of the radius vector $\underline{r}$ with respect to the principal axes are

$$
\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\eta_x & s\eta_x \\ 0 & -s\eta_x & c\eta_x \end{bmatrix} \begin{bmatrix} c\eta_t & 0 & s\eta_t \\ 0 & 1 & 0 \\ -s\eta_t & 0 & c\eta_t \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}
$$

$$
= - \begin{bmatrix} s\eta_t \\ s\eta_x c\eta_t \\ c\eta_x c\eta_t \end{bmatrix} \tag{7}
$$

where η_x is the elevation angle of the sun vector from the orbital plane and η_t is the orbital angle from midnight (definitions in Appendix A). For zero attitude error the torque about the vehicle axes becomes

$$\underline{T}_{GP} = 3\Omega^2 \begin{bmatrix} \Delta I_x s\eta_x c\eta_x c^2\eta_t \\ \Delta I_y c\eta_x s\eta_t c\eta_t \\ \Delta I_z s\eta_x s\eta_t c\eta_t \end{bmatrix} \tag{8}$$

or

$$\underline{T}_{GP} = \frac{3}{2}\Omega^2 \begin{bmatrix} \frac{1}{2} s2\eta_x (1 + c2\eta_t) \Delta I_x \\ c\eta_x s2\eta_t \Delta I_y \\ s\eta_x s2\eta_t \Delta I_z \end{bmatrix} \tag{9}$$

If the attitude of the vehicle is held constant between η_{ta} and η_{tb} the following momenta will accumulate (where $\underline{H}_o$ is the stored momentum at η_{ta}, and the components are in the P-system).

$$\underline{H} = \underline{H}_o + \frac{3}{4}\Omega \begin{bmatrix} \frac{1}{2} s2\eta_x \left[(2\eta_{tb} + s2\eta_{tb}) - (2\eta_{ta} + s2\eta_{ta})\right] \Delta I_x \\ - c\eta_x (c2\eta_{tb} - c2\eta_{ta}) \Delta I_y \\ - s\eta_x (c2\eta_{tb} - c2\eta_{ta}) \Delta I_z \end{bmatrix}$$

ANGULAR MOMENTUM DESATURATION METHOD

The assumed inertia distribution for case two is such that ΔI_x and ΔI_y are almost equal in magnitude, whereas ΔI_z is very small. Equation (9) shows that only the x-axis has a bias torque. It is comparable in magnitude with the cyclic y-torque. The only way to desaturate the momentum caused by the bias torque is by reversing the sign on the sun elevation angle η_x; i.e., a large maneuver about the x-axis at the beginning and at the end of the night is mandatory. Therefore, this maneuver is part of any desaturation method for case two [1, 5]. Visualization of the gravity gradient torque (which, for the general case, has been attempted in Reference 6) for case two is relatively simple when the large moments of inertia are assumed to be equal. The torque is then perpendicular to the gravity gradient and the z_p-axis (minimum moment-of-inertia axis), and it tends to align the latter with the former. Then, the gravity gradient desaturation method is developed in an inertially fixed orbital coordinate system, I, where the x_I-axis is in the orbital plane and perpendicular to the sun vector, the y_I-axis is perpendicular to the orbital plane pointing north, and the z_I-axis completes the right-handed coordinate system. The z_I-axis coincides with the projection of the sun vector into the orbital plane.

The x-momentum desaturation is the most critical because, for maximum momentum accumulation on the day-half of the orbit ($|\eta_x| = 45$ degrees), instantaneous maneuvers of 90 degrees about the x-axis would be required at the terminators to equal the accumulation. Clearly, more capability is required; therefore, other maneuvers for x-desaturation are necessary. The best that can be done is to keep the z_p-axis at a 45-degree angle with respect to the gravity gradient direction (maximum gravity gradient torque) and have the projection of the z_p-axis into the orbital plane coincide with the gravity gradient direction. Under these circumstances the x_I-torque is proportional to the cosine of the orbital angle, η_t, rather than the square of the cosine, resulting in a much larger desaturation capability than the maximum day accumulation. The z_p-axis then describes a cone in inertial space. A constant y_I-torque results when the z_p-axis projection does not coincide with the direction of the gravity gradient but is offset by a constant angle. This is used for the y-momentum desaturation. A z-momentum results only when the integration interval is not symmetrical with respect to midnight. Nonsymmetry is therefore used for the z-momentum desaturation. The desaturation method also has to work when an x-desaturation (sun vector in the orbital plane) is unnecessary and it was found that a tilting of the maneuver cone by a small z_I-rotation will result in the desired desaturation capability as shown in the equations developed later.

A series of single-axis maneuvers is initiated at the beginning of the night to arrive at the desired attitude. Constant angular velocities about two axes are required to hold this constant attitude of the minimum moment-of-inertia axis with respect to the direction of the gravity gradient. Another series of single-axis maneuvers brings the vehicle back to the desired sun-oriented day attitude at the end of the night. The series of maneuvers (always about the instantaneous vehicle axes) is given in Table 1.

TABLE 1. SINGLE-AXIS MANEUVERS

At Beginning of Night		Before End of Night	
Axis	Amount	Axis	Amount
x	$-\eta_x$	x	$-\mu_x$
z	$+\mu_z$	y	$-\Delta\eta_t - \mu_y - \Delta\mu_y$
y	$+\Delta\eta_t + \mu_y - \Delta\mu_y$	z	$-\mu_z$
x	$+\mu_x$	x	$+\eta_x$

The two y-maneuvers require further explanation. They are performed in the orbital plane (for $\mu_z = 0$) which is illustrated in Figure 1. The projection of the z_p-axis moves through an angle of $2\Delta\mu_y$ while the z_p-axis is coning between the two sets of single-axis maneuvers. The coning requires an inertial angular velocity of the vehicle about the y_I-axis which is resolved by μ_x into the y_p- and z_p-axes.

All maneuvers are performed by applying precalculated constant angular velocities about the appropriate vehicle axes for precalculated time intervals, identified by the equivalent orbital angle η_t in Table 2.

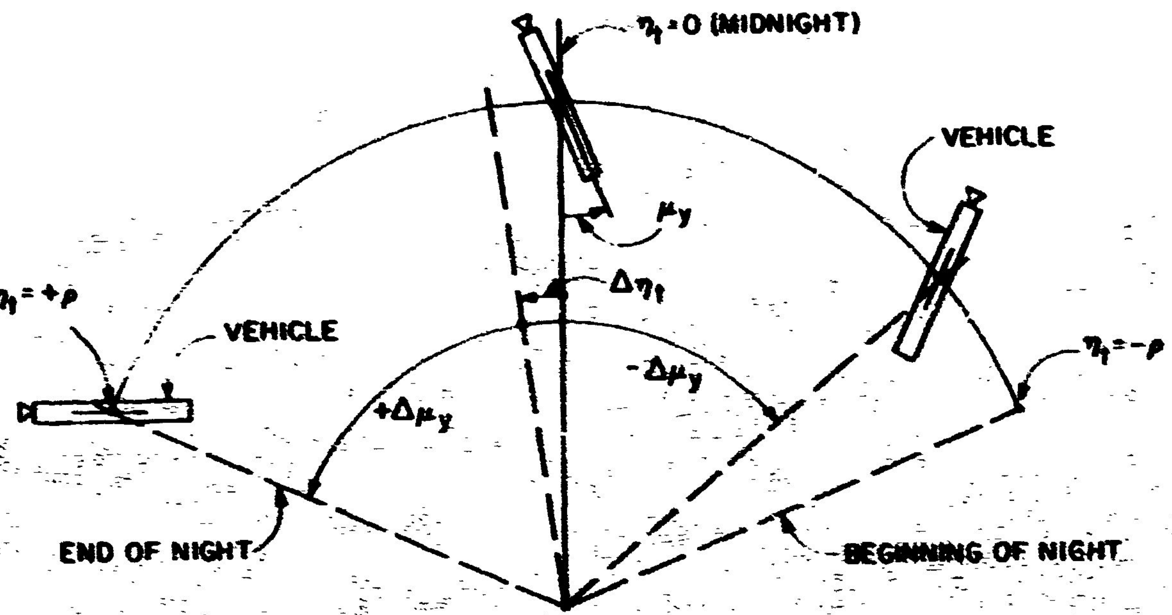


Figure 1. Maneuver angles.

TABLE 2. ANGULAR VELOCITY COMMANDS FOR x, y, AND z VEHICLE AXES

x	$\dot{\epsilon}_{x1}$	0	0	$\dot{\epsilon}_{x2}$	0	$\dot{\epsilon}_{x3}$	0	0	$\dot{\epsilon}_{x4}$
y	0	0	$\dot{\epsilon}_{y1}$	0	$\dot{\epsilon}_{y2}$	0	$\dot{\epsilon}_{y3}$	0	0
z	0	$\dot{\epsilon}_{z1}$	0	0	$\dot{\epsilon}_{z2}$	0	0	$\dot{\epsilon}_{z3}$	0

η_{t1} η_{t2} η_{t3} η_{t4} η_{t5} η_{t6} η_{t7} η_{t8} η_{t9} η_{t10}

where

$$\dot{\epsilon}_{x1} = -\dot{\phi}_L \operatorname{sgn} \eta_x$$

$$\dot{\epsilon}_{x2} = +\dot{\phi}_L \operatorname{sgn} \mu_x$$

$$\dot{\epsilon}_{x3} = -\dot{\phi}_L \operatorname{sgn} \mu_x$$

$$\dot{\epsilon}_{x4} = + \dot{\phi}_L \operatorname{sgn} \eta_x$$

$$\dot{\epsilon}_{y1} = - \dot{\phi}_L$$

$$\dot{\epsilon}_{y2} = + \Omega\ c\mu_x$$

$$\dot{\epsilon}_{y3} = - \dot{\phi}_L$$

$$\dot{\epsilon}_{z1} = + \dot{\phi}_L \operatorname{sgn} \mu_z$$

$$\dot{\epsilon}_{z2} = - \Omega\ s\mu_x$$

$$\dot{\epsilon}_{z3} = - \dot{\phi}_L \operatorname{sgn} \mu_z$$

and

$$\eta_{t1} = \eta_{t2} - K_\epsilon |\eta_x|$$

$$\eta_{t2} = \eta_{t3} - K_\epsilon |\mu_z|$$

$$\eta_{t3} = \eta_{t4} - K_\epsilon |\mu_y + \Delta\eta_t - \Delta\mu_y|$$

$$\eta_{t4} = \eta_{t5} - K_\epsilon |\mu_x|$$

$$\eta_{t5} = \Delta\eta_t - \Delta\mu_y$$

$$\eta_{t6} = \Delta\eta_t + \Delta\mu_y$$

$$\eta_{t7} = \eta_{t6} + K_\epsilon |\mu_x|$$

$$\eta_{t8} = \eta_{t7} + K_\epsilon |\mu_y + \Delta\eta_t + \Delta\mu_y|$$

$$\eta_{t9} = \eta_{t8} + K_\epsilon |\mu_z|$$

$$\eta_{t10} = \eta_{t9} + K_\epsilon |\eta_x| \quad ,$$

with

$$K_\epsilon = \Omega/\dot{\phi}_L \quad , \tag{11}$$

where Ω is the orbital rate and $\dot{\phi}_L$ is the angular rate limit of the vehicle about the large moment-of-inertia axis.

DESATURATION COMMANDS

Each desaturation method can be split into two parts. One part is concerned with the generation of the desaturation commands; the other part is concerned with the scheme to desaturate the commanded momentum. These two parts are rather independent of each other; e.g., several different methods for the momentum command generation could use the same method for the maneuver angle generation.

Several facts that affect the momentum command generation must be born in mind. The first is that approximations will have to be made for the calculation of the angular momentum desaturated by a given set of maneuvers, or the mathematical treatment becomes unmanageable. Secondly, the stored angular momentum is the difference between the accumulation and the desaturation and does not indicate the desaturation itself. Furthermore, the assumptions were made that the large inertias are equal and that there is no misalignment between the principal moment-of-inertia axes and the geometric vehicle axes. It is therefore impossible to generate an exact command, and the desaturation command has to be updated by a summation process such that $(i = x, y, z)$

$$H_{Din} = H_{Di(n-1)} + \Delta H_{Di} \tag{12}$$

where H_{Din} is the present desaturation command, $H_{Di(n-1)}$ is that of the past orbit and ΔH_{Di} is the desired change. The latter has to be chosen such that H_{Di} will home in on the exact solution. Sample data system consideration shows the following form to be adequate [3,4]:

$$\Delta H_{Din} = K_n H_{Ain} + K_{(n-1)} H_{Ai(n-1)} \quad ; \tag{13}$$

i.e., it is necessary to also consider the past orbit. H_{Ai} is an average angular momentum dependent on the stored momentum of the following form:

$$H_{Ai} = \frac{1}{2} (H_{imax} + H_{imin}) - H_{Bi} \quad . \tag{14}$$

This form averages the peak momenta (during the day-half of the orbit only) and provides a selectable offset through $H_{\bar{B}}$ which allows the conversion, on the average, to any point in the momentum space.

For ease of initialization (initial values for the angles are known, and the ones for the momentum commands are not) equation (12) is replaced by equations of the form

$$\mu_{in} = \mu_{i(n-1)} + K_{ij} \Delta H_{Dj} \tag{15}$$

The "memory" is now in the past maneuver commands rather than in the summation of the desaturation command changes; cf. equation (12).

DESATURATION MANEUVER EFFECTIVENESS

The effectiveness of the commanded maneuver angles μ_x, μ_y, μ_z, $\Delta\mu_y$, and $\Delta\eta_t$ has to be established if the desired momentum is to be desaturated. The angle $\Delta\mu_y$ will always be large; μ_x can be large. But μ_y, μ_z, and $\Delta\eta_t$ will be less than [illegible] degrees and small angle approximations can be assumed to simplify the development; i.e., second-order and higher-order terms are neglected. The same reasoning allows neglecting any product of the small angles.

Equation (6) shows the gravity gradient torque about body axes in the P-system. The components of $\underline{r}$ in the P-system are

$$[\text{illegible}] \tag{16}$$

or

$$\begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} = \begin{bmatrix} +\mu_y \\ +\mu_z c\mu_x s\eta_t - s\mu_x \\ -\mu_z s\mu_x s\eta_t - c\mu_x \end{bmatrix} \tag{17}$$

Transformation of the gravity gradient torque to the I-system results in

$$\underline{T}_{GI} = \begin{bmatrix} 1 & -\mu_z & 0 \\ \mu_z & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\eta_t & 0 & s\eta_t \\ 0 & 1 & 0 \\ -s\eta_t & 0 & c\eta_t \end{bmatrix} \begin{bmatrix} 1 & 0 & \mu_y \\ 0 & 1 & 0 \\ -\mu_y & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\mu_x & -s\mu_x \\ 0 & s\mu_x & c\mu_x \end{bmatrix} \underline{T}_{GP} \tag{18}$$

Evaluation of equation (18) with the use of equations (6) and (17) results in (second-order terms are neglected; details are in Appendix B)

$$\underline{T}_{GI} = \frac{3\Omega^2 \Delta I_x}{2} \begin{bmatrix} s2\mu_x \, c\eta_t - \mu_z \, c2\mu_x \, s2\eta_t \\ \mu_y (1 + c2\mu_x) + \mu_z \, s2\mu_x \, c\eta_t \\ -s2\mu_x \, s\eta_t + \mu_z \, c2\mu_x (1 - c2\eta_t) \end{bmatrix} \tag{19}$$

This torque has to be integrated over the orbital angle interval from $\eta_{t5} = \Delta\eta_t - \Delta\mu_y$ to $\eta_{t6} = \Delta\eta_t + \Delta\mu_y$ (Table 2 and Fig. 1). The accumulated momentum components in the I-system (inertially fixed orbital midnight system) are (details are in Appendix B)

$$H_x = \frac{2}{A_1} s2\mu_x \, s\Delta\mu_y \tag{20}$$

$$H_y = \frac{2}{A_1} \left[\mu_y (1 + c2\mu_x) \Delta\mu_y + \mu_z \, s2\mu_x \, s\Delta\mu_y \right] \tag{21}$$

$$H_z = \frac{2}{A_1} \left[-\Delta\eta_t \, s2\mu_x \, s\Delta\mu_y + \frac{1}{2} \mu_z \, c2\mu_x (2\Delta\mu_y - s2\Delta\mu_y) \right] \tag{22}$$

where

$$A_1 = \frac{2}{3\Omega \, \Delta I_x}$$

Expressions of the maneuver angles as a function of the commanded momentum are needed such that the actual momentum of equations (20), (21), and (22) equals or approaches the commanded momentum. Additional constraints are needed since there are more maneuver angles than equations.

The x-momentum desaturation takes first priority and equation (20) shows that $\Delta\mu_y$ should be as large as possible, subject to the available maneuver time. It will therefore be made a function of all the other maneuver angles and is not considered independent; cf. equation (34).

Equations (12) and (15) show that there is interest in the gains connected with the change of the maneuver commands. Partial differentiation will supply the one for ΔH_x:

$$\partial H_x = \frac{2}{A_1} \, 2c2\mu_x \, s\Delta\mu_y \, \partial\mu_x$$

or

$$\frac{\partial\mu_x}{\partial H_x} = \frac{A_1}{4s\Delta\mu_y \, c2\mu_x} \tag{23}$$

A small positive quantity, k, has to be added to $c2\mu_x$ to avoid the possibility for infinite change of μ_x, which is only an indication of the lack of effectiveness of μ_x. The μ_x-value of the previous orbit can be used for $c2\mu_x$ since μ_x changes slowly. The x-gain now has the form

$$K_{xx} = \frac{A_1}{4s\Delta\mu_y \, (c2\mu_x + k)} \tag{24}$$

The problem has been reduced to equations (21) and (22) and three maneuver angles (μ_y, μ_z, and $\Delta\eta_t$). The capability of μ_y to desaturate a y-momentum is relatively large, and it is therefore logical to address the z-momentum desaturation first assuming that μ_y can easily compensate for any effect of μ_z on H_y.

Equation (22) shows that the effectiveness of μ_z on H_z is proportional to $c2\mu_x$ and that of $\Delta\eta_t$ is proportional to $s2\mu_x$. This suggests the following relations

$$\mu_z = \frac{A_1\, c2\mu_x}{2\Delta\mu_y - s2\Delta\mu_y}\, H_{Dz} \tag{25}$$

$$\Delta\eta_t = -\frac{A_1\, s2\mu_x}{2s\Delta\mu_y}\, H_{Dz} \tag{26}$$

Elimination of μ_z and $\Delta\eta_t$ from equation (22) with the aid of equations (25) and (26) shows that the actual momentum, H_z, equals the commanded momentum, H_{Dz}. The relationships of equations (25) and (26) also hold for the changes of the angles for a change in the momentum command and we have

$$K_{zz} = \frac{A_1\, c2\mu_x}{2\Delta\mu_y - s2\Delta\mu_y} \tag{27}$$

$$K_{\Delta z} = -\frac{A_1\, s2\mu_x}{2s\Delta\mu_y} \tag{28}$$

With μ_z known, the actual and the commanded y-momentum can now be made identical by

$$\mu_y = \frac{A_1}{2\Delta\mu_y\,(1 - c2\mu_x)}\,H_{Dy} - \frac{s2\mu_x\, s\Delta\mu_y}{\Delta\mu_y\,(1 - c2\mu_x)}\,\mu_z \qquad (29)$$

which results in the gains

$$K_{yy} = \frac{A_1}{2\Delta\mu_y\,(1 + c2\mu_x)} \qquad (30)$$

$$K_{yz} = -\frac{s2\mu_x\, s\Delta\mu_y}{\Delta\mu_y\,(1 - c2\mu_x)} \quad . \qquad (31)$$

When the above equations are cast into the form of equation (15) we have

$$\mu_{xn} = \mu_{x(n-1)} + K_{xx}\,\Delta H_{Dx}$$

$$\mu_{yn} = \mu_{y(n-1)} + K_{yy}\,\Delta H_{Dy} + K_{yz}\,K_{zz}\,\Delta H_{Dz}$$

$$\mu_{zn} = \mu_{z(n-1)} + K_{zz}\,\Delta H_{Dz}$$

$$\Delta\eta_{tn} = \Delta\eta_{t(n-1)} + K_{\Delta z}\,\Delta H_{Dz}$$

with

$$K_{xx} = \frac{A_1}{4s\Delta\mu_y\,(c2\mu_x - k)}$$

$$K_{yy} = \frac{A_1}{2\Delta\mu_y\,(1 + c2\mu_x)}$$

$$K_{yz} = -\frac{2s2\mu_x \, s\Delta\mu_y}{2\Delta\mu_y \, (1 + c2\mu_x)}$$

$$K_{zz} = \frac{c2\mu_x \, A_1}{2\Delta\mu_y - s2\Delta\mu_y}$$

$$K_{\Delta z} = -\frac{s2\mu_x \, A_1}{2s\Delta\mu_y} \quad . \qquad (32)$$

The angular momenta are measured about the vehicle axes (which, under the assumption of no misalignment, is the P-system) while being sun-oriented, but for the development so far, all momenta components were assumed to be in the I-system, and a resolution is necessary:

$$(\Delta H)_I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\eta_x & -s\eta_x \\ 0 & s\eta_x & c\eta_x \end{bmatrix} (\Delta H)_P \qquad (33)$$

Assume an orbital angle from $-\rho$ to $+\rho$ (usually equivalent to the night interval) with respect to midnight is available for desaturation: then $\Delta\mu_y$ is

$$\Delta\mu_y = \rho - K_\epsilon \left(\eta_x + \mu_z + \Delta\mu_y + \mu_y + \Delta\eta_t - \mu_x \right) \quad . \qquad (34)$$

This calculation will be made after the gains K_{ij} are established; i.e., the gain calculation is performed with the $\Delta\mu_y$ value of the past orbit and the new $\Delta\mu_y$ is calculated with the maneuver angles of the present orbit; cf. equations (28) through (34). The reasoning is that the K_{ij}'s are only an approximation, whereas a correct $\Delta\mu_y$ is important for maximum efficiency of the desaturation method.

CONCLUSION AND RECOMMENDATIONS

The minimum percentage of the orbit needed for full angular momentum desaturation using gravity gradient torques is 31.9 percent, where the following assumptions are made: the sunline elevation angle η_X is 45 degrees (worst case), the vehicle has infinite angular rate capability, and no other torques besides gravity gradient act on the vehicle. The normalized torque profile about the x_I-axis for this case is shown in Figure 2. A finite angular rate causes an increase of the minimum desaturation percentage. For example, simulation shows that the minimum desaturation percentage increases to about 38 percent for a maximum rate of 0.01 rad/s. Consideration of other external torques (aerodynamic, magnetic, etc.) could further increase the minimum desaturation percentage.

A satellite in a circular orbit with an altitude of 435 km, for example, has a nighttime of only 33.4 percent for η_X = 45 degrees, and the conclusion must be drawn that valuable daytime which should be available for experiments is needed for momentum desaturation. Unless other means can be utilized for momentum desaturation (magnetic torques, etc.), the recommendation is to review the mission as to whether it is possible to have the minimum moment-of-inertia axis perpendicular to the sunline and simultaneously in the orbital plane (for case one, see Reference 3). Such an orientation results in very small desaturation percentages, especially if it is possible to keep the difference between the large moments of inertia small. Angular velocity requirements for case-one vehicles are about a magnitude smaller than those for case-two vehicles, allowing the use of a smaller capacity for the momentum storage system.

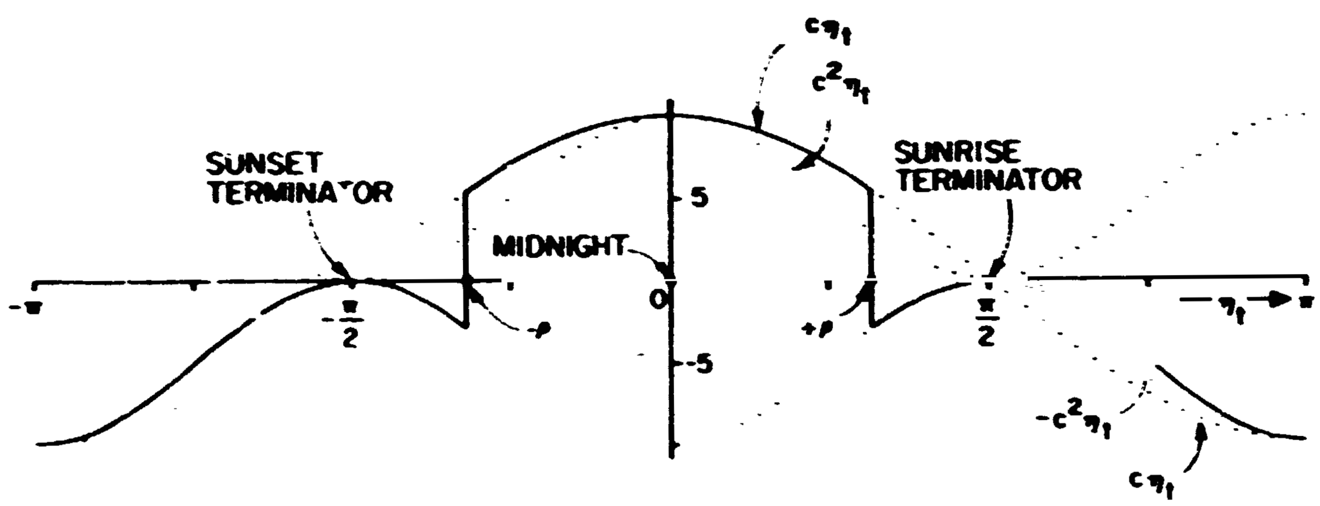


Figure 2. Normalized x_I-torque profile.

APPENDIX A

DEFINITIONS OF ATM COORDINATE SYSTEMS AND ANGLES

The angles and coordinate systems used for the ATM are shown in Figures A-1 through A-6. All systems were not necessary for the development of the desaturation method. (The ones not used do not appear in the DEFINITION OF SYMBOLS.)

Symbols	Transformation Matrix	Definition
X_E Y_E Z_E		The equatorial system is the basic system to which all others are referenced. It is an earth-centered inertial frame with the z-axis toward vernal equinox and the y-axis toward earth north (Fig. A-2).
X_O Y_O Z_O	$X_O = [A_z][A_y]X_E$	The orbital system is an earth-centered frame with the z-axis toward the ascending node and the y-axis toward orbital north, identified as a quasi-inertial system (Fig. A-3).
X_R Y_R Z_R	$X_R = [\eta_x][\eta_y]X_O$	The reference system provides an ideal reference for the vehicle. It has the z-axis toward the sun center and the x-axis in the orbital plane identified as a quasi-inertial system (Fig. A-5).
X_V Y_V Z_V	$X_V = [\theta]X_R$	The vehicle system is geometrically centered in the ATM LM CSM with the z-axis toward the ATM.
X_P Y_P Z_P	X_P = [illegible] X_V	The principal axis system is similar to the vehicle system except that the origin is at the vehicle center of mass and the axes are along the principal moment-of-inertia axes.

Symbols	Transformation Matrix	Definition
X_D Y_D Z_D	$X_D = [\eta_t + \eta_y] X_O$	The disturbance system in which aerodynamic and gravity gradient forces are defined. It rotates in the orbital plane with the vehicle and maintains the z-axis toward earth center. The x-axis is tangent to the orbit and opposite to the velocity vector (circular orbit only) (Fig. A-6).
X_S Y_S Z_S	$X_S = [\Phi_z] X_E$	The sun or ecliptical system is an earth-centered inertial frame with the z-axis toward the vernal equinox and the x-axis in the plane of the ecliptic (Fig. A-4).
X_I Y_I Z_I	$X_I = [\eta_y] X_O$	The intermediate system is an orbital coordinate system between the O-system and the R-system. The x-axis is perpendicular to the sunline and in the orbital plane. The y-axis is perpendicular to the orbital plane pointing north.

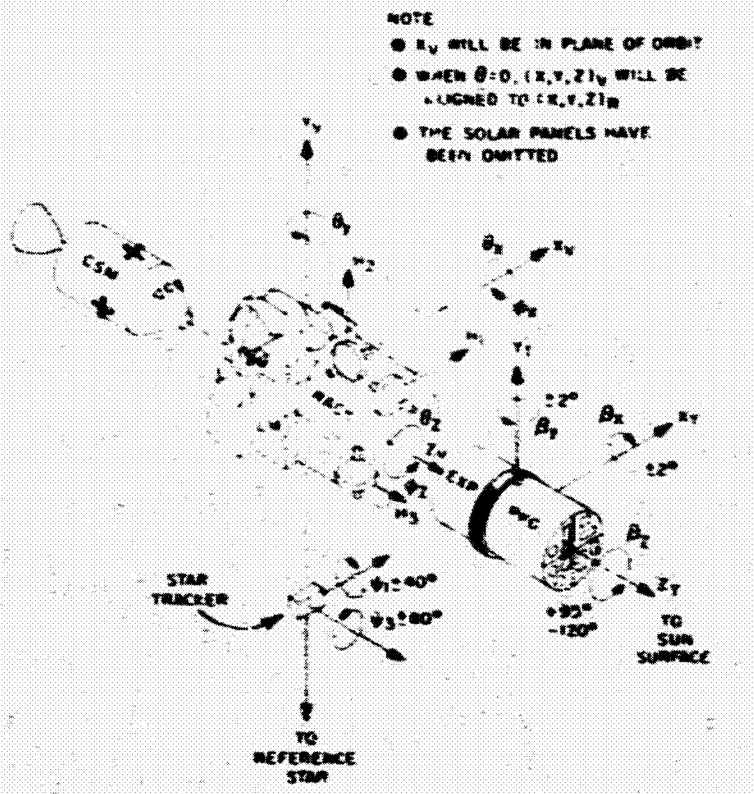
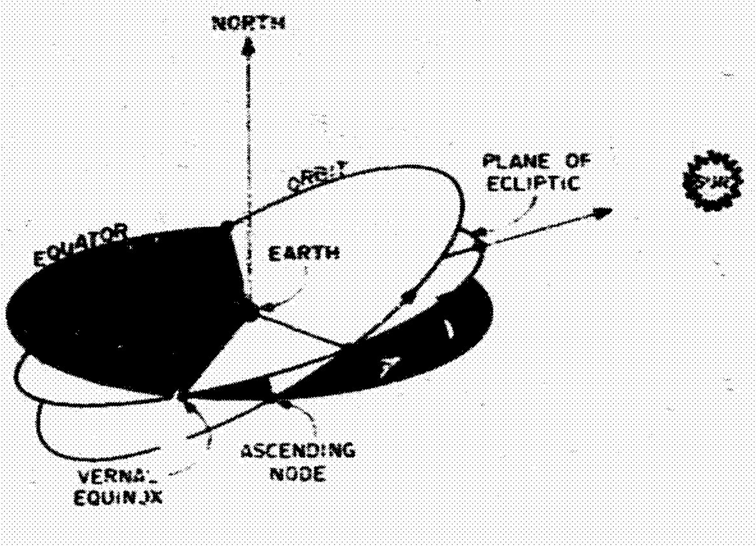


Figure A-1. ATM/LM/CSM configuration.



THESE THREE PLANES FURNISH THE BASIC REFERENCE FOR DEVELOPMENT OF ATM COORDINATE SYSTEMS

Figure A-2. The celestial sphere.

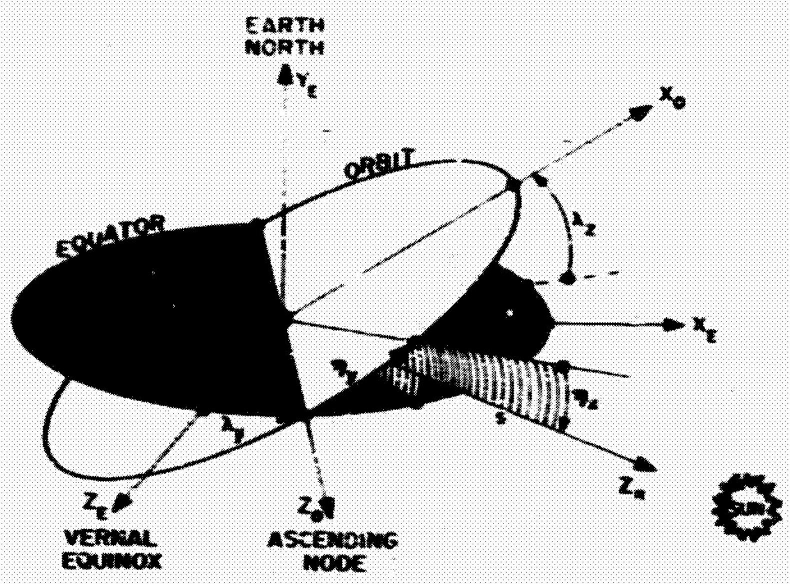


Figure A-3. ATM orbital coordinate system relative to equatorial system $X_O = [\lambda_z][\lambda_y]X_E$.

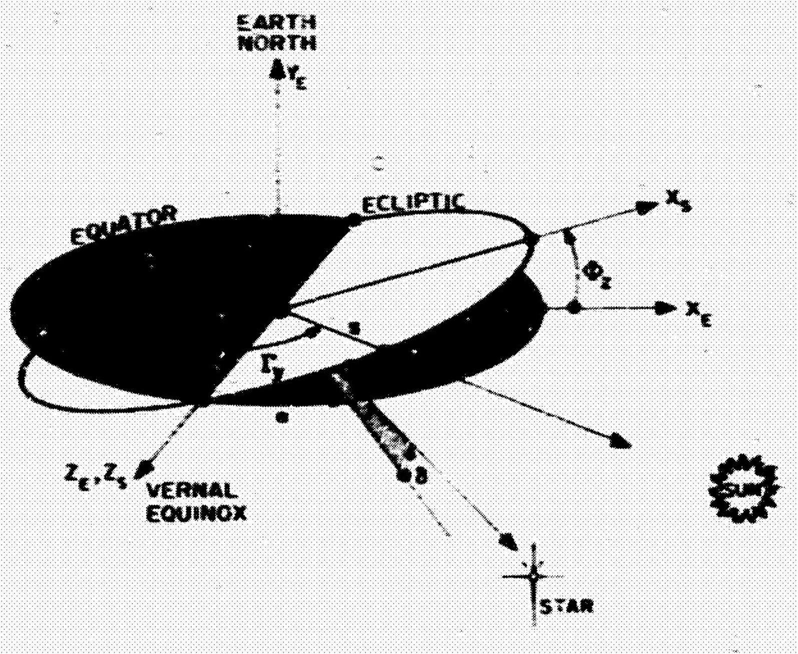


Figure A-4. ATM sun (ecliptic) coordinate system relative to equatorial system $X_S = [\Phi_z]X_E$.

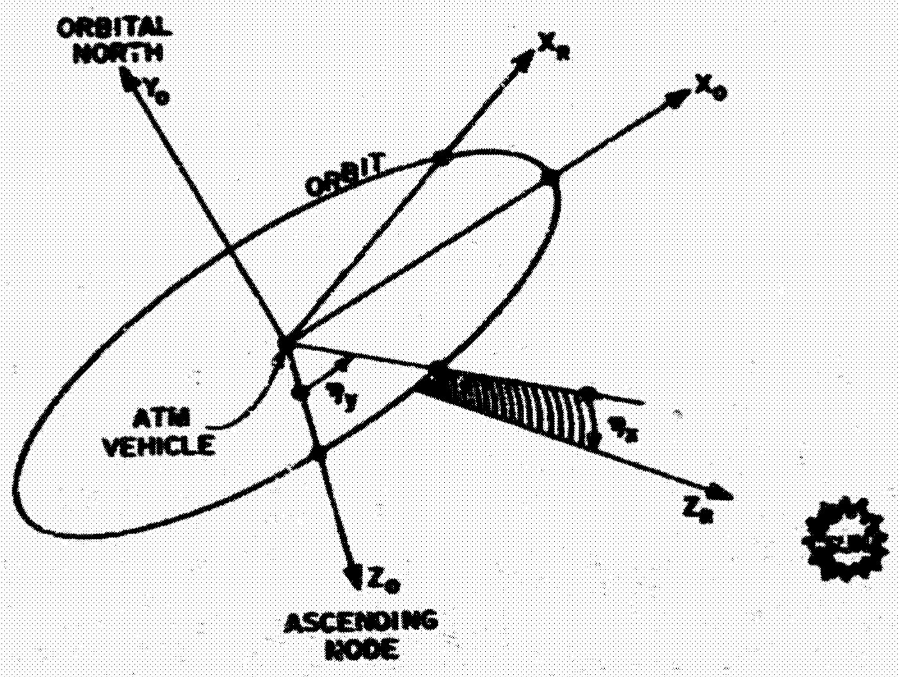


Figure A-5. ATM reference coordinate system relative to orbital system $X_R = [\eta_x][\eta_y]X_O$.

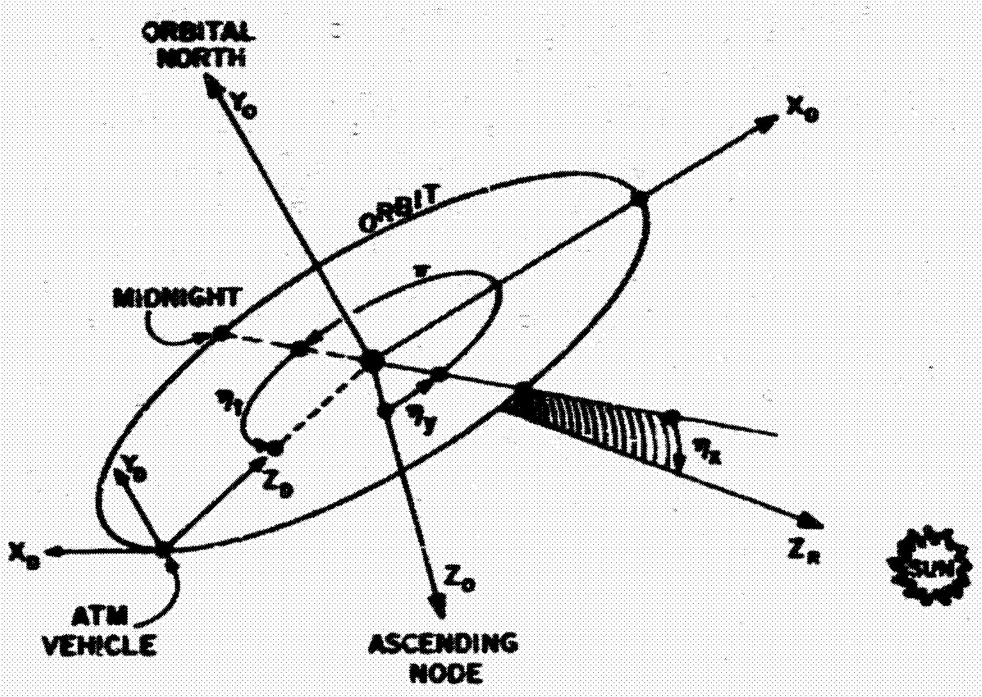


Figure A-6. ATM disturbance coordinate system relative to orbital system $X_D = [\eta_t + \eta_y]X_O$.

APPENDIX B

DESATURATION TORQUE AND ANGULAR MOMENTUM COMPONENTS

The components of the desaturation torque should be in an inertial frame for ease of integration to obtain the angular desaturation momentum. The components of the desaturation torque in the T-system are [see equation (18)]:

$$\underline{T}_{GT} = \begin{bmatrix} 1 & -\mu_z & 0 \\ \mu_z & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\eta_t & 0 & s\eta_t \\ 0 & 1 & 0 \\ -s\eta_t & 0 & c\eta_t \end{bmatrix} \begin{bmatrix} 1 & 0 & \mu_y \\ 0 & 1 & 0 \\ -\mu_y & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\mu_x & -s\mu_x \\ 0 & s\mu_x & c\mu_x \end{bmatrix} \underline{T}_{GP} \; .$$

Evaluation and use of equation (6) leads to

$$\underline{T}_{GT} = 3\Omega^2 \Delta I_x \, r_z \begin{bmatrix} c\eta_t(r_y - \mu_y r_x s\mu_x) - s\eta_t(\mu_y r_y + r_x s\mu_x) + \mu_z r_x c\mu_x \\ \mu_z \left[c\eta_t(r_y - \mu_y r_x s\mu_x) - s\eta_t(\mu_y r_y + r_x s\mu_x) \right] - r_x c\mu_x \\ -s\eta_t(r_y - \mu_y r_x s\mu_x) - c\eta_t(\mu_y r_y + r_x s\mu_x) \end{bmatrix}$$

or, with equation (17) and neglecting second-order terms,

$$\underline{T}_{GT} = \frac{3}{2} \Omega^2 \Delta I_x \begin{bmatrix} s2\mu_x \, s\eta_t - \mu_z \, c2\mu_x \, s2\eta_t \\ \mu_y(1 + c2\mu_x) + \mu_z \, s2\mu_x \, c\eta_t \\ -s2\mu_x \, s\eta_t + \mu_z \, c2\mu_x (1 - c2\eta_t) \end{bmatrix}$$

Integration of the x-component of $\underline{T}_{GT}$ from

$$\eta_{t5} = \Delta\eta_t - \Delta\mu_y \quad \text{to} \quad \eta_{t6} = \Delta\eta_t + \Delta\mu_y$$

results in (with $A_1 = 2 \; 3\Omega \, \Delta I_x$):

$$H_x A_1 = s2\mu_x \int_{\eta_{t5}}^{\eta_{t6}} c\eta_t \, d\eta_t - \mu_z \, c2\mu_x \int_{\eta_{t5}}^{\eta_{t6}} s2\eta_t \, d\eta_t$$

$$= s2\mu_x \, (s\eta_{t6} - s\eta_{t5}) + \mu_z \, c2\mu_x \frac{1}{2} \, (c2\eta_{t6} - c2\eta_{t5}) \quad .$$

Substitution of η_{t5} and η_{t6} and neglecting second-order effects results in

$$H_x = \frac{2}{A_1} \, s2\mu_x \, s\Delta\mu_y \quad .$$

Similarly for the y-component we have

$$H_y A_1 = \mu_y \, (1 + c2\mu_x) \int_{\eta_{t5}}^{\eta_{t6}} d\eta_t + \mu_z \, s2\mu_x \int_{\eta_{t5}}^{\eta_{t6}} c\eta_t \, d\eta_t$$

$$= \mu_y \, (1 + c2\mu_x) \, (\eta_{t6} - \eta_{t5}) + \mu_z \, s2\mu_x \, (s\eta_{t6} - s\eta_{t5}) \quad .$$

After substitution of the values for η_t we have

$$H_y = \frac{2}{A_1} \left[\mu_y \, (1 + c2\mu_x) \, \Delta\mu_y + \mu_z \, s2\mu_x \, s\Delta\mu_y \right] \quad .$$

Development of the z-component results in

$$H_z A_1 = - s2\mu_x \int_{\eta_{t5}}^{\eta_{t6}} s\eta_t \, d\eta_t + \mu_z \, c2\mu_x \int_{\eta_{t5}}^{\eta_{t6}} (1 - c2\eta_t) \, d\eta_t$$

$$= - s2\mu_x \, 2\Delta\eta_t \, s\Delta\mu_y + \mu_z \, c2\mu_x \, (2\Delta\mu_y - s2\Delta\mu_y) \quad .$$

After substitution of the values for η_t we have

$$H_z = \frac{2}{\Lambda_1}\left[-\Delta\eta_t\, s2\mu_x\, s\Delta\mu_y + \frac{1}{2}\,\mu_z\, c2\mu_x\,(2\Delta\mu_y - s2\Delta\mu_y)\right] \quad .$$

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LARGE ANGLE METHOD FOR SPACE VEHICLE ANGULAR MOMENTUM DESATURATION USING GRAVITY GRADIENT TORQUES

By Hans F. Kennel

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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